

PROBABILISTIC STRUCTURAL ANALYSIS METHODS  
FOR SELECT SPACE PROPULSION SYSTEM  
STRUCTURAL COMPONENTS\*

T. A. CRUSE  
P. O. DRAWER 28510  
SOUTHWEST RESEARCH INSTITUTE  
SAN ANTONIO, TEXAS 78284

ABSTRACT

The paper presents a summary of the status of this five-year project which is now in its third year of research and development. The goal of the project is the development of several methodologies for probabilistic structural modeling. Probabilistic structural modeling consists of stochastic models of material properties, part geometries, boundary conditions, as well as loading conditions. The current presentation focuses on one methodology — coupling of an advanced finite element structural analysis code with probabilistic modeling strategies. The essential algorithm developments for combining the finite element and probabilistic analysis methods are reported. The validity of the resulting probabilistic structural analysis method is confirmed through a series of test problems with exact results based on Monte Carlo simulations. Additionally, the applicability of the method to a Space Propulsion System (a turbine blade) is demonstrated for static stresses.

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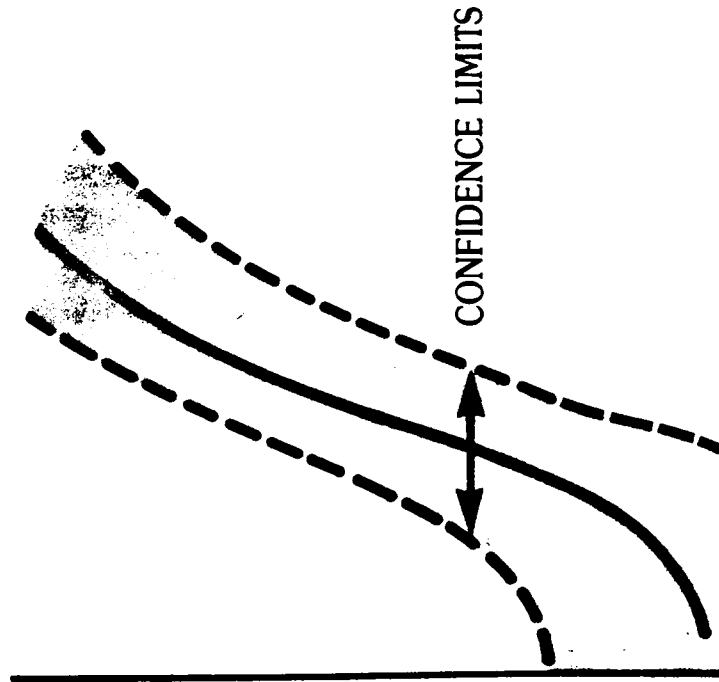
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# PROBABILISTIC DESIGN METHODS WILL SIMULATE "REAL WORLD" STRUCTURAL RESPONSES

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CUMULATIVE PROBABILITY



DUE TO DESIGN UNCERTAINTIES:

- LOADING
- MATERIAL BEHAVIOR
- GEOMETRY, TOLERANCES
- BOUNDARY CONDITIONS

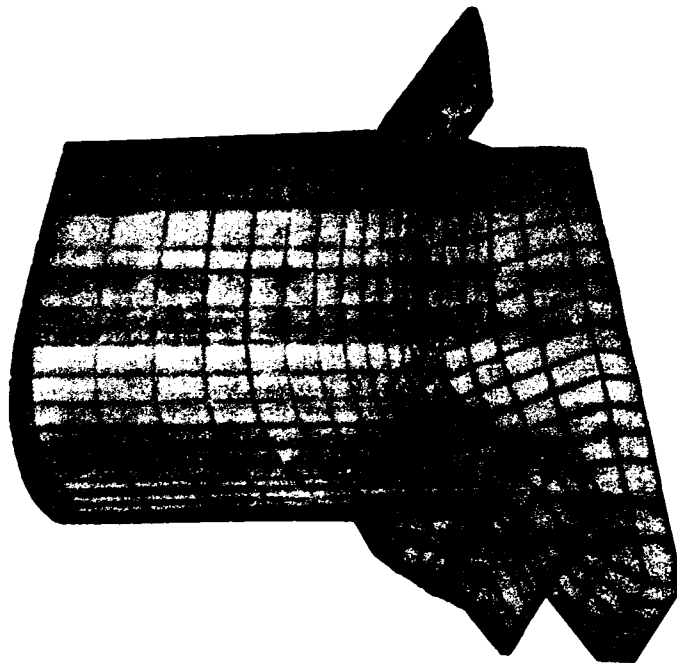


THE GOAL OF THE PSAM PROJECT IS TO PROVIDE THE STRUCTURAL ANALYST WITH THE ABILITY TO SIMULATE "REAL WORLD" PROBLEMS WHERE MANY OF THE KEY VARIABLES ARE SUBJECT TO DESIGN VALUE UNCERTAINTIES. SPECIFICALLY, PSAM WILL PROVIDE A COMPLETE CUMULATIVE DISTRIBUTION FUNCTION OF USER DEFINED OUTPUT VARIABLES OVER A SPECIFIED RANGE OF PROBABILITY LEVELS. FOR CASES WHERE THE UNCERTAINTIES ARE BASED ON LIMITED EXPERIMENTAL DATA, PSAM WILL ALSO ESTIMATE CONFIDENCE BANDS ON THE CDF RESULTS.

# **RELIABILITY ESTIMATION METHODS INTEGRATED WITH FINITE ELEMENT ANALYSIS**

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- USER DEFINES UNCERTAIN DATA IN MODEL
- FEM MODELS PREDICT DESIGN SENSITIVITY
- RELIABILITY METHODS COMBINE UNCERTAINTY AND SENSITIVITY DATA
  - FAST PROBABILITY INTEGRATION (FPI)
  - MONTE CARLO SIMULATION (MC)



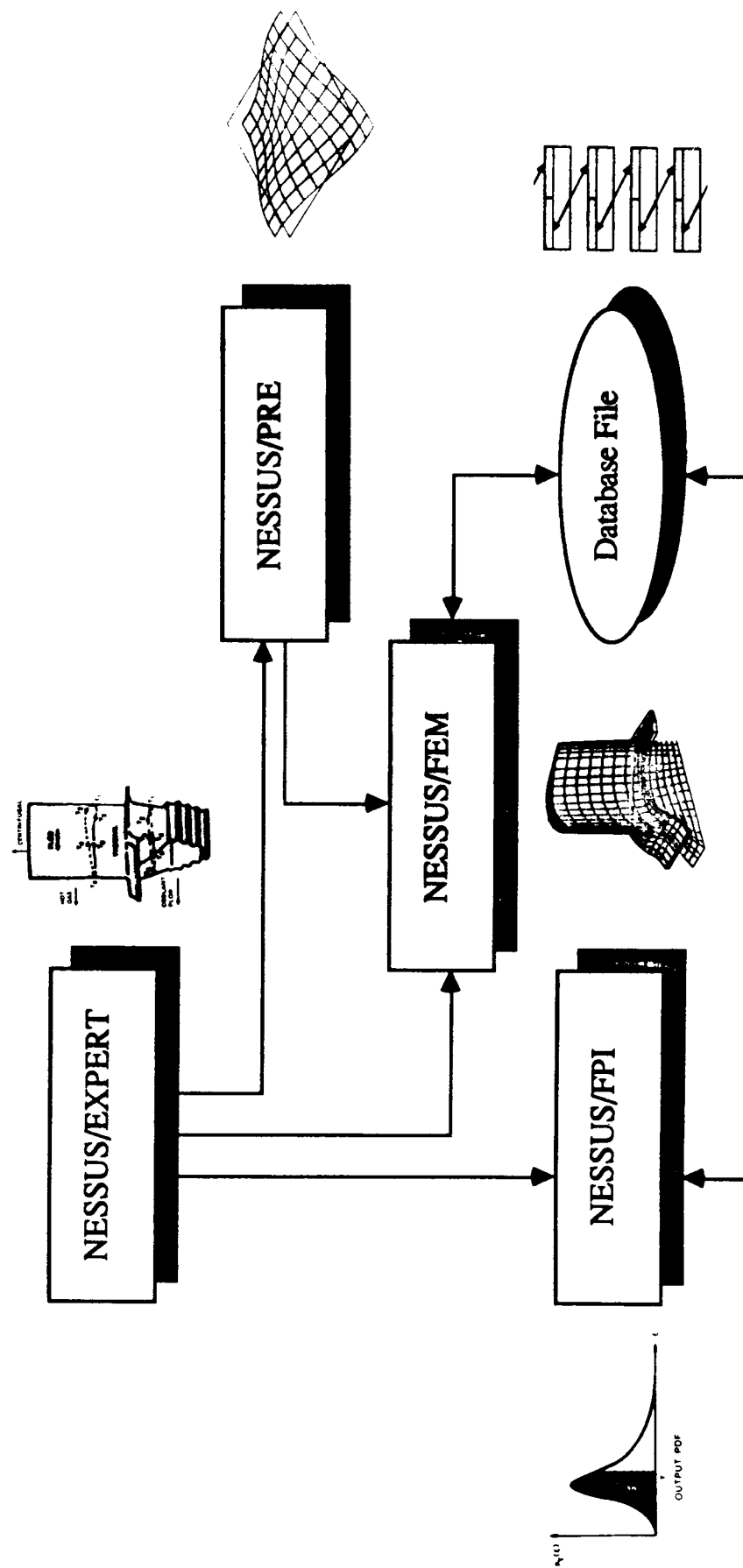
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THE FIRST PROBABILISTIC STRUCTURAL ANALYSIS TOOL IN THE PSAM SYSTEM IS THE FINITE ELEMENT METHOD. A MEANS FOR COMBINING THE FEM WITH PROBABILISTIC METHODS HAS BEEN ESTABLISHED. THE FEM IS USED TO ESTABLISH THE SENSITIVITIES OF THE STRUCTURAL ANALYSIS RESULT TO CHANGES IN USER-DEFINED RANDOM VARIABLES. ADVANCED RELIABILITY METHODS FOR PROBABILISTIC CALCULATIONS COMBINE THE FEM-GENERATED SENSITIVITIES WITH USER-DEFINED STATISTICS FOR THE INPUT RANDOM VARIABLES. TWO PROBABILITY INTEGRATION ALGORITHMS ARE COMBINED WITH THE FEM MODULE: (1) FAST PROBABILITY INTEGRATION (FPI); AND (2) AN ENHANCED MONTE CARLO SIMULATION.

# AN OVERVIEW OF THE NESSUS CODE

## Five Major Software Modules Working Together



THE COMBINATION OF FEM AND FPI ALGORITHMS IS ACCOMPLISHED WITHIN A MODULAR PACKAGE CALLED NESSUS (NONLINEAR EVALUATION OF STOCHASTIC STRUCTURES UNDER STRESS). THE MODULES INCLUDE EXPERT (THE USER INTERFACE), PRE (DEFINES INDEPENDENT RANDOM VARIABLES FOR PARTIALLY CORRELATED RANDOM FIELDS--E.G., PRESSURES AND TEMPERATURES), FEM, FPI, AND A SPECIAL DATA BASE. THE FEM, FPI, AND EXPERT MODULES WILL BE DISCUSSED.

# **NESSUS/FEM CODE DEVELOPED FOR PROBABILISTIC MODELING**

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## **NODAL SOLUTION STRATEGY**

- ALL INPUT DATA
- EQUILIBRIUM ITERATION
- ALL OUTPUT DATA

## **USER INTERFACE STRATEGY**

- KEYWORD DATA STRUCTURE
- INTERFACE TO NESSUS/PRE
- DATABASE TRANSLATORS
- EXPERT SYSTEM

## **GENERAL SOLUTION CAPABILITY**

- LINEAR, NONLINEAR
- STATIC, DYNAMIC
- IN-CORE SOLUTION
- MULTIPLE ELEMENT TYPES
- RANDOM LOADS
- RANDOM VARIABLES

## **ADVANCED ELEMENT FORMULATIONS**

- SURFACE NODE BASED
- HYBRID SHELL/PLATE
- ENHANCED SOLID
- SPECIAL THERMAL LOADS

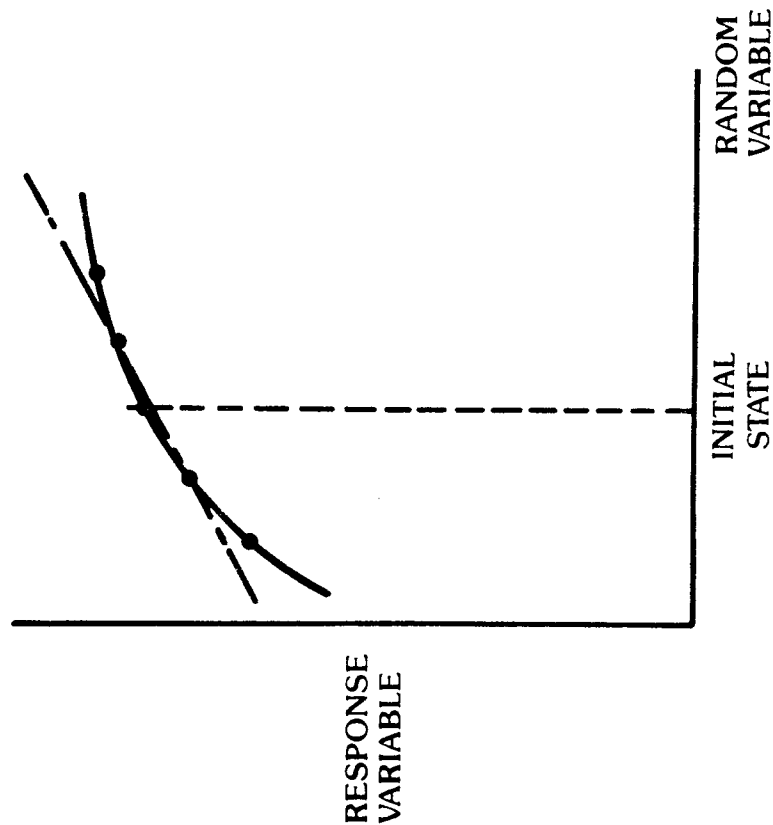




A SPECIAL FEM CODE HAS BEEN DEFINED FOR NESSUS. THE FEM CODE IS BASED ON NODAL VARIABLES AND NODAL EQUILIBRIUM (USING A MIXED VARIATIONAL METHOD). THE FEM ANALYSIS INCLUDES LINEAR, NONLINEAR, STATIC, DYNAMIC, AND TRANSIENT CAPABILITIES. RANDOM VARIABLES INCLUDE THOSE SHOWN. THE USER INTERFACE IS BASED ON A KEYWORD DATA STRUCTURE WITH INTERFACES TO PRE AND EXPERT. THE ELEMENTS IN THE FEM MODULE INCLUDE STANDARD ELEMENTS PLUS AN ASSUMED STRAIN 8-NODE SOLID ELEMENT AND AN ASSUMED STRESS 16-NODE SOLID ELEMENT. BOTH THE ELEMENTS ARE TO HAVE HIGH ASPECT RATIOS AND SEVERE THERMAL LOAD CAPABILITIES.

# NESSUS/FEM GENERATES RESPONSE MODEL BY EFFICIENT PERTURBATION ANALYSIS

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- INDEPENDENT RANDOM VARIABLES
  - GEOMETRY
  - MATERIAL PROPERTIES
  - BOUNDARY CONDITIONS
- ITERATIVE SOLUTION ALGORITHM
  - SMALL PERTURBATIONS
  - RETAIN INITIAL STIFFNESS MATRIX
  - MODIFY RIGHT-HAND SIDE ONLY
- RESPONSE SURFACE FITTING
  - LEAST SQUARES ERROR
  - LINEAR OR QUADRATIC



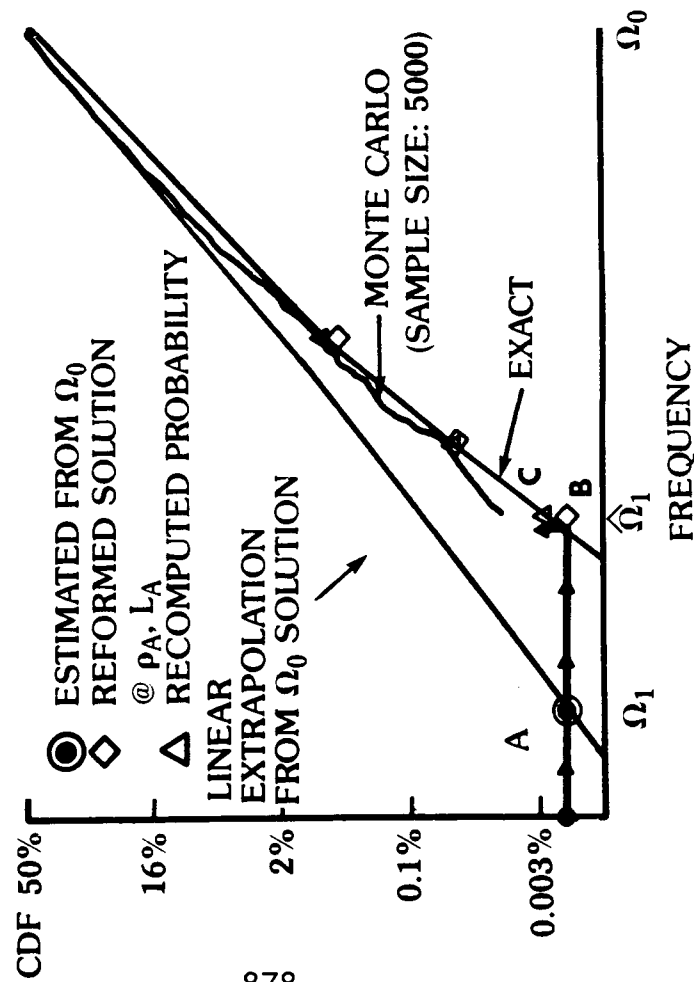
THE PERTURBATION OF THE FEM ANALYSIS TO DETERMINE SOLUTION SENSITIVITY TO THE RANDOM VARIABLES HAS BEEN TREATED BY A SPECIAL ALGORITHM. FOR SMALL PERTURBATIONS, THE FEM ANALYSIS USES ITERATION WITH THE NON-PERTURBED REDUCED STIFFNESS MATRIX AS A PRE-CONDITIONER MATRIX. THE EFFECTS OF RANDOM VARIABLES CHANGES ARE TRANSFERRED TO THE RIGHT-HAND SIDE OF THE SYSTEM EQUATIONS. THE SOLUTION STATE (RESPONSE SURFACE) IS LEAST-SQUARES FITTED TO THE PERTURBED SOLUTIONS AS A HYPERPLANE OR A QUADRATIC SURFACE.



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THE FPI ALGORITHM IS DEMONSTRATED IN A SAMPLE PROBLEM OF VIBRATION OF A CANTILEVER BEAM. TWO RANDOM VARIABLES (DENSITY, LENGTH) ARE NORMALIZED TO THEIR MEAN VALUES ( $\mu$ ) AND STANDARD DEVIATION ( $\sigma$ ). AN EXACT SOLUTION SURFACE FOR THE VALUE OF NATURAL FREQUENCY FOR THE DETERMINISTIC STATE IS TAKEN TO BE  $\Omega_0$ , SHOWN AS A CARTOON-CURVE DEPENDING ON  $\rho, L$ . A LINEAR APPROXIMATION TO THIS EXACT RESPONSE SURFACE IS SHOWN. IF  $\rho, L$  ARE ARBITRARILY TAKEN TO BE GAUSSIAN DISTRIBUTIONS, THE JOINT PROBABILITY LEVELS ARE CIRCLES, AS SHOWN. IF WE DESIRE TO ESTIMATE THE PROBABILITY OF EXCEEDING A NATURAL FREQUENCY  $\hat{\Omega}_1$  ( $< \Omega_0$ ), THE PROCESS INVOLVES A SIMPLE INTERACTION ALGORITHM. THE LINEAR FIT AT  $\Omega_0$  IS USED TO ESTABLISH THE  $\rho, L$  SURFACE FOR  $\hat{\Omega}_1$ . THE FPI CODE RAPIDLY CALCULATES THE VOLUME UNDER THE JOINT-PDF SURFACE FOR POINTS BEYOND THE SHIFTED LINEAR APPROXIMATION--CORRESPONDING TO POINT A IN FIGURE 8. DUE TO SOLUTION NONLINEAR DEPENDENCE ON  $\rho, L$ , THE ACTUAL NATURAL FREQUENCY AT A IS  $\Omega_1$ , SHOWN AS POINT B IN FIGURE 8. THE VALUE  $\hat{\Omega}_1$  IS OBTAINED BY A REFORMULATION SOLUTION STEP ON THE FEM MODEL. A NEW TANGENT PLANE TO THE RESPONSE SURFACE FOR  $\Omega_1$  IS OBTAINED BY PERTURBATION OF THE FEM MODEL. FPI THEN CALCULATES A REVISED PROBABILITY ESTIMATE--POINT C IN FIGURE 8. THE RESULT IS SEEN TO HAVE CONVERGED TO THE EXACT SOLUTION IN THESE TWO STEPS.

# **ITERATION ALGORITHM ACHIEVES ACCURATE PROBABILITY ESTIMATES**

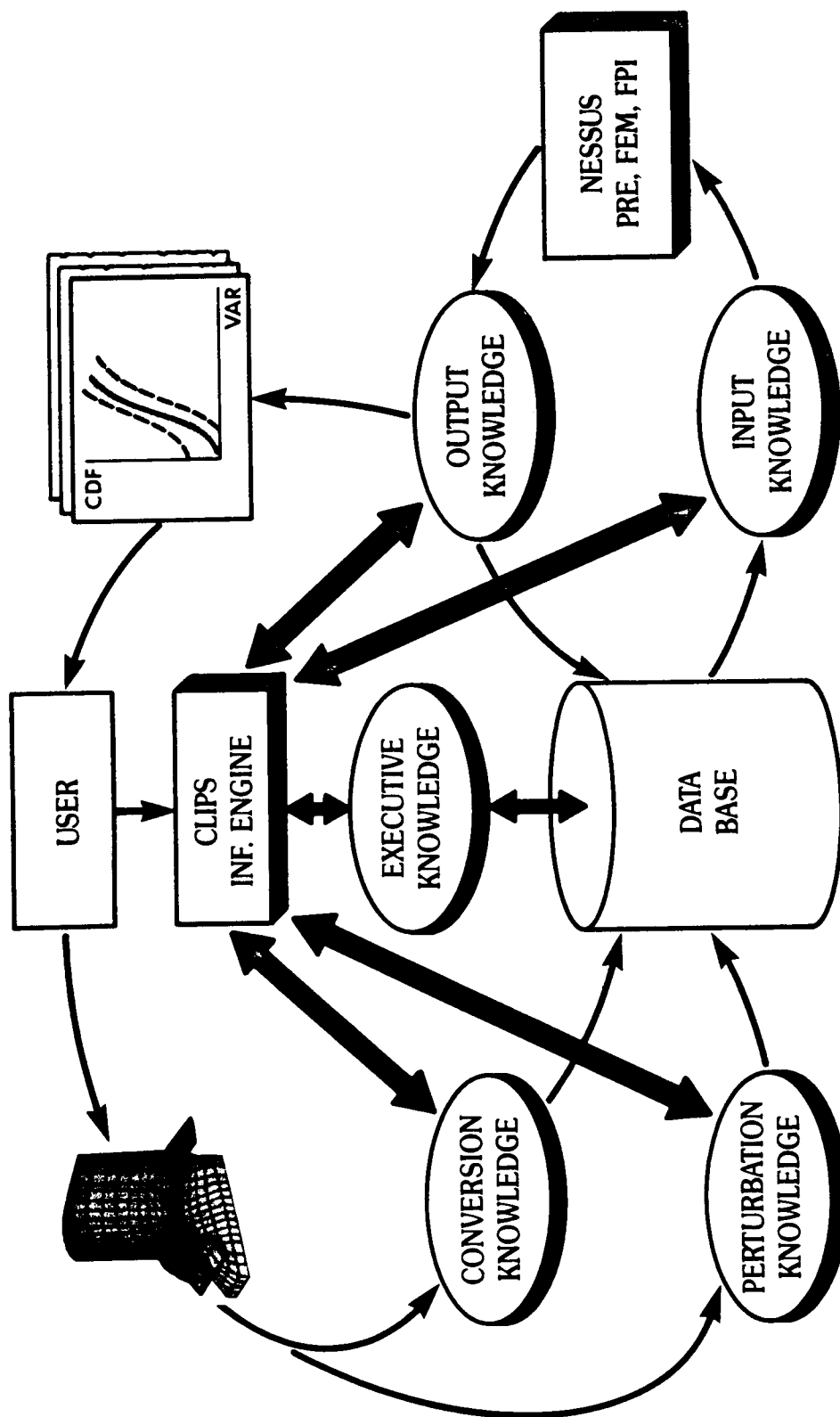


- EXTRAPOLATES RANDOM VARIABLES FROM APPROXIMATE SOLUTIONS
- REFORMS SOLUTION AT NEW RANDOM VARIABLE VALUES
- RECALCULATES PROBABILITY FROM LOCAL APPROXIMATION
- TESTS ON CONVERGENCE



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# EXPERT KNOWLEDGE WILL GOVERN USER INTERFACE



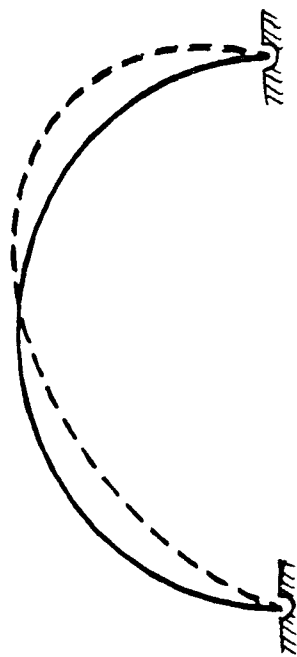
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THE USER INTERFACE IS BEING ESTABLISHED USING AN EXPERT SYSTEM SUPPORTED BY A VARIETY OF FORTRAN SUBROUTINES. THE EXPERT SHELL IS CLIPS, WRITTEN AT NASA/JSC IN C-LANGUAGE. EXPERT KNOWLEDGE IS BEING INCORPORATED IN CLIPS-RULES TO GOVERN EACH OF THE MAJOR ITEMS IN A PROBABILISTIC STRUCTURAL ANALYSIS. AS AN EXAMPLE, THERE ARE RULES REGARDING ELEMENT TYPES AND PERTURBATION SIZES THAT CAN BE USED TO ALLOW CLIPS TO DEFINE PERTURBATION SIZES SUCH THAT A FORTRAN SUBROUTINE AUTOMATICALLY CREATES THE FORTRAN DATA SET CORRESPONDING TO THE PERTURBATION.

## NESSUS ASSESSES BUCKLING SENSITIVITY

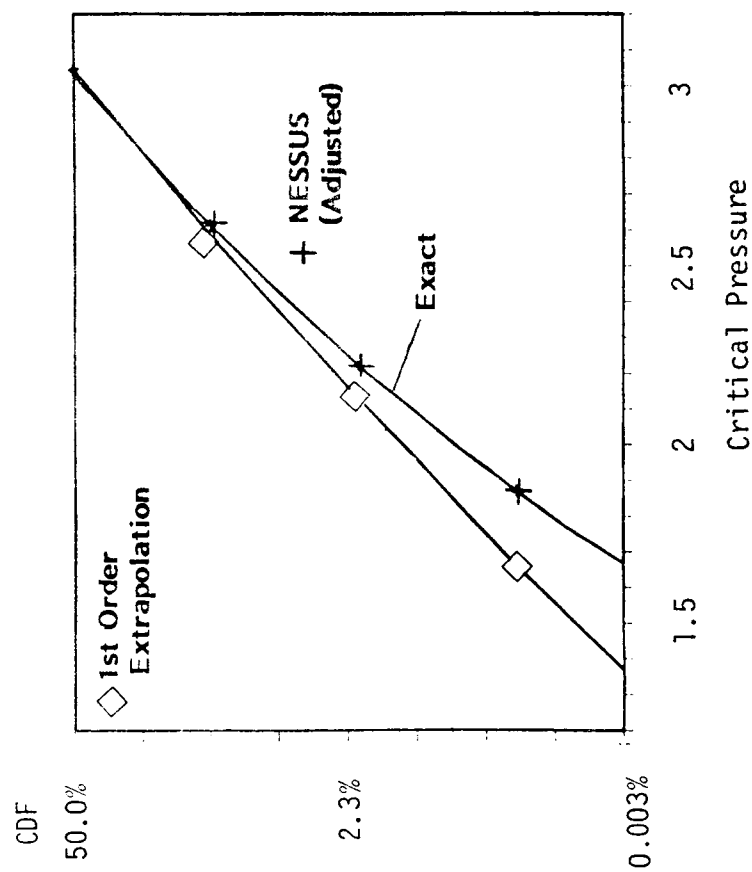


### First Shell Buckling Mode

- Pinned ends
- Infinite length
- Uniform pressure
- 10% error in mean solution
- NESSUS 75 (4-node flat shell)

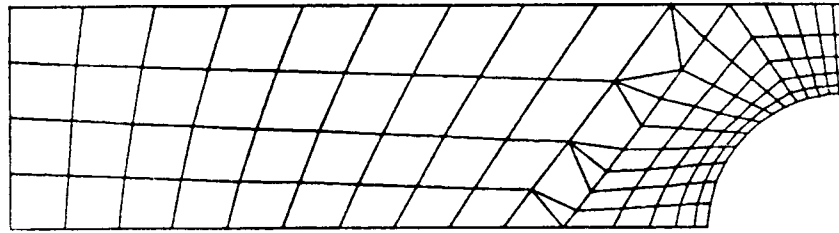
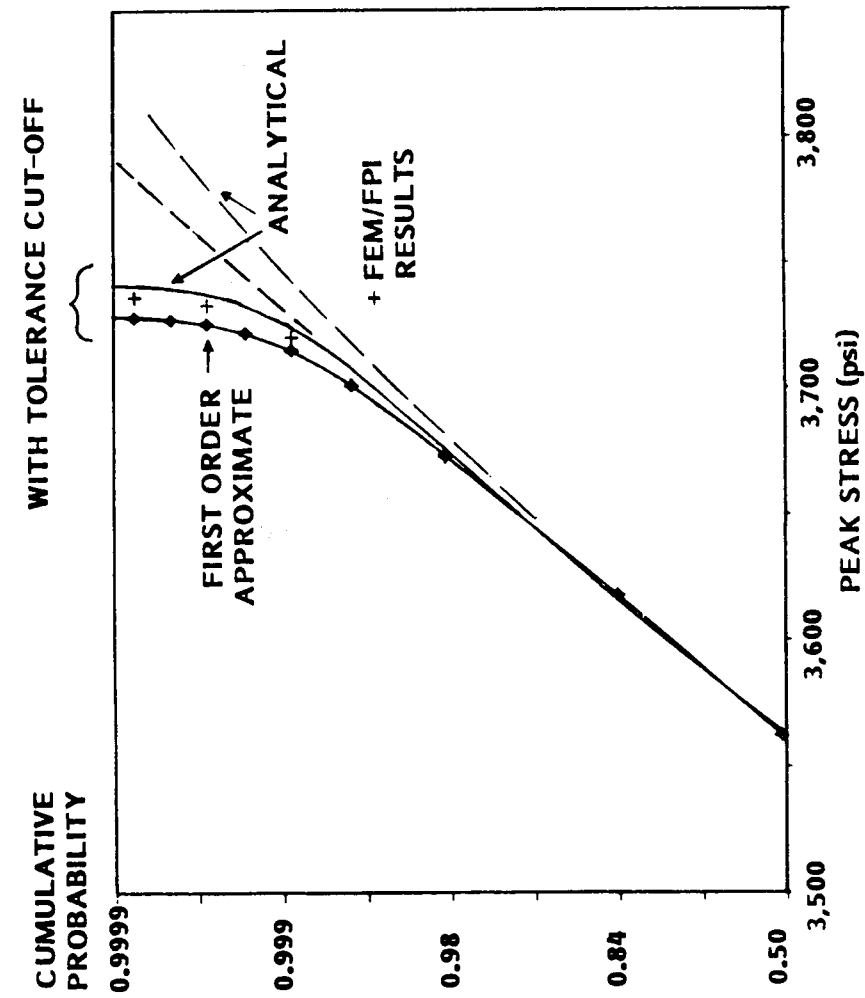
### Probabilistic Analysis

- Thickness uncertainty
- Log normal distribution
- 5% COV



THE NEXT NESSUS VALIDATION PROBLEM CONCERNS PREDICTION OF THE CRITICAL BUCKLING PRESSURE OF A CYLINDRICAL SHELL WITH AN UNCERTAIN SHELL THICKNESS, FOR THE INDICATED BUCKLING MODE. THE SOLUTION PARAMETERS ARE GIVEN ALONG WITH THE STATISTICAL INPUT DATA. THE SOLUTION WAS ESTABLISHED AT THREE PROBABILITY LEVELS SHOWN BY THE DATA POINTS. THE MVFO SOLUTION CORRESPONDS TO THE A-SOLUTIONS ON FIGURE 8. THE X-DATA POINTS CORRESPOND TO THE B-SOLUTIONS (NO C-SOLUTION ITERATION REQUIRED). CORRESPONDENCE TO THE EXACT SOLUTION WAS EXCELLENT, BASED ON A CALIBRATED SHIFT OF THE PREDICTED BUCKLING PRESSURES, BASED ON A 10% ERROR IN THE MEAN (DETERMINISTIC) SOLUTION. THUS WE SEE THAT THE NESSUS ALGORITHM IS ABLE TO ACCURATELY PREDICT DISTRIBUTIONS EVEN THOUGH THERE WILL ALWAYS BE MODELING ERRORS IN THE DETERMINISTIC SOLUTION.

# NESSUS FEM/FPI MODELS TOLERANCE EFFECTS ON KT



## LOCAL STRESSES

- o 2D Plane Stress
- o Uniform Tension
- o Uncertain Radius
  - $\pm 2\%$
  - $\pm 6\%$  Cut-off
  - Normal Distribution

THIS VALIDATION PROBLEM DEMONSTRATES THE USE OF A TRUNCATED DISTRIBUTION TO DESCRIBE ONE OF THE INPUT VARIABLES--THE NOTCH RADIUS. THE FINITE ELEMENT MODEL SIMULATES THE EFFECT OF GEOMETRIC UNCERTAINTY ON THE PEAK STRESS CONCENTRATION. THE GAUSSIAN DISTRIBUTION OF NOTCH SIZE UNCERTAINTY WAS TRUNCATED AT  $\pm 6\%$  VARIATION TO SIMULATE TOLERANCE LIMITS. THE TRUNCATION ON THE MAXIMUM STRESS IS CLEARLY SHOWN.

# TURBINE BLADE ANALYSIS

- RANDOM VARIABLES

- GEOMETRY
- MATERIAL ORIENTATION
- MATERIAL PROPERTIES

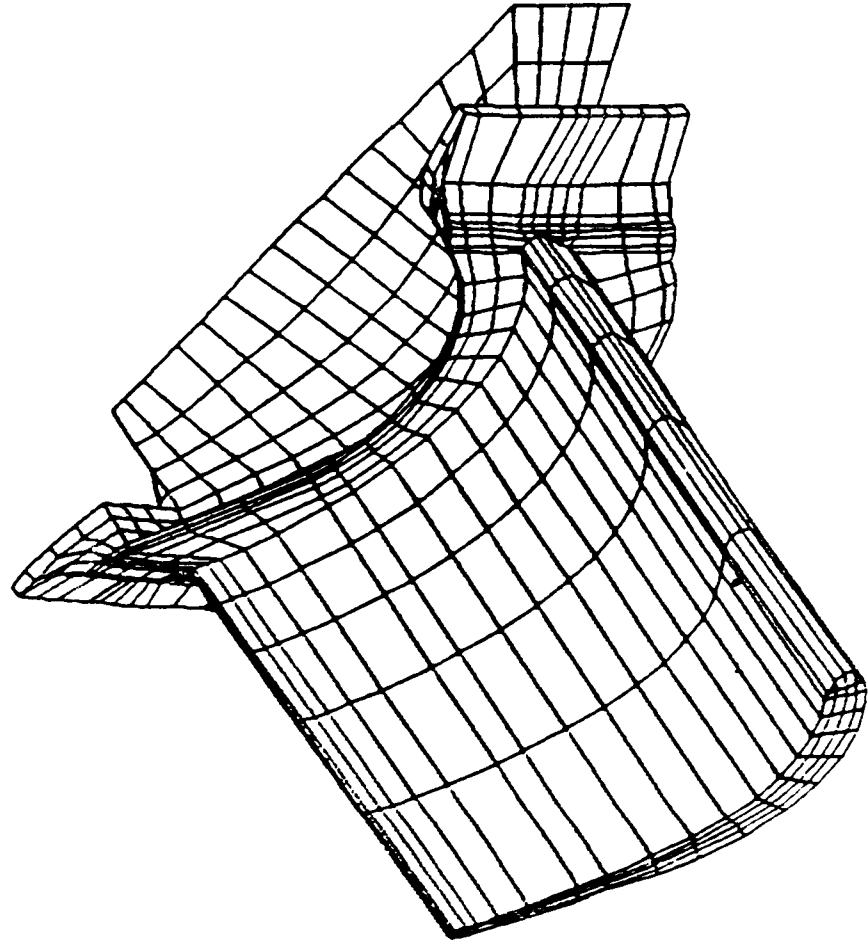
- STEADY STATE

- CENTRIFUGAL LOAD
- TEMPERATURE
- STATIC PRESSURE AND  $\Delta P$

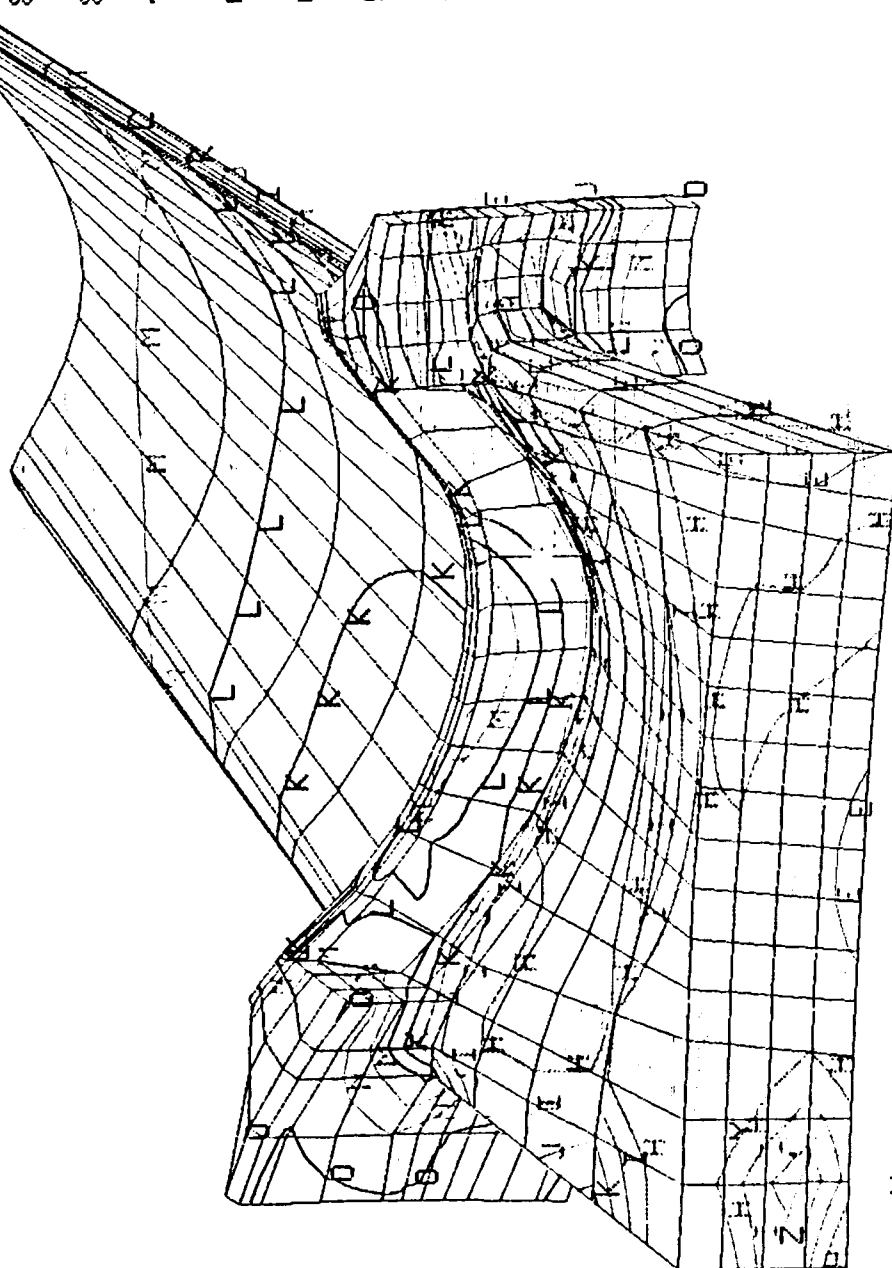
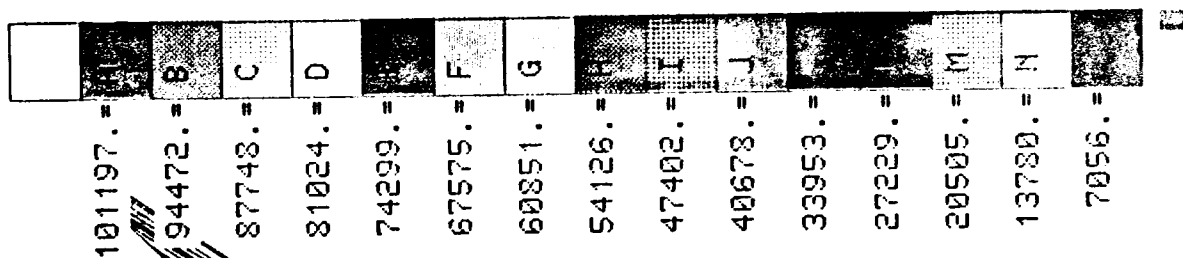
- RESPONSE VARIABLES

- CENTRIFUGAL STRESS
- PRESSURE STRESS
- FREQUENCY

- 3-D SOLID MODEL



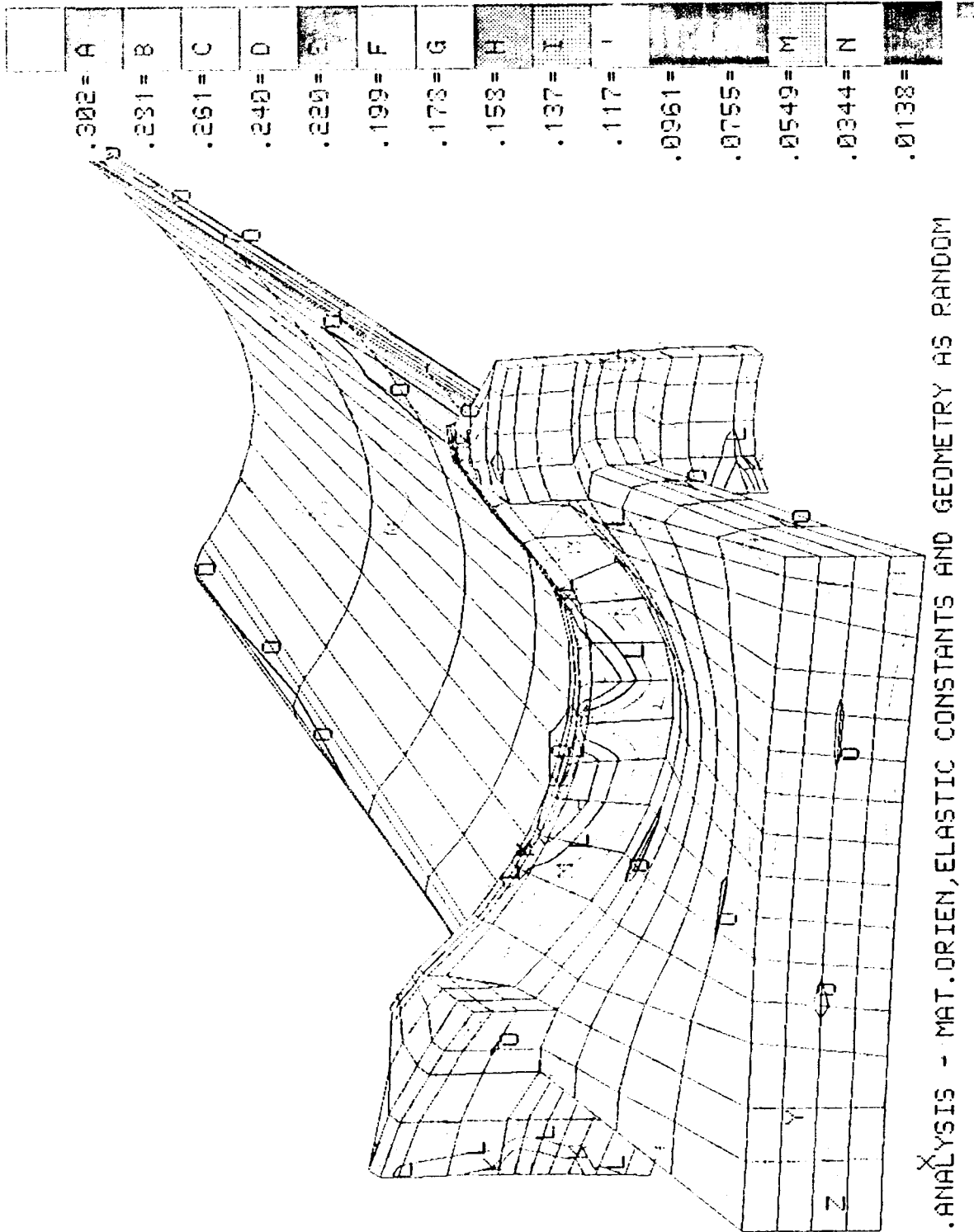
SOME NUMERICAL RESULTS FOR THE FIRST VERIFICATION PROBLEM FOR PSAM ARE TO BE GIVEN. THE MODEL RESULTS ARE FOR THE SSME TURBOPUMP BLADES. THE FEM MODEL OF THE TURBINE BLADE HAS ABOUT 2,500 NODES (1,456 ELEMENTS) AND 6,000 DEGREES OF FREEDOM USING EIGHT-NODE ISOPARAMETRIC SOLID ELEMENTS. THE STEADY-STATE LOAD RESPONSE IS ANALYZED WITH BLADE GEOMETRY (TWIST, TILT), SINGLE-CRYSTAL AXIS ORIENTATIONS (PRIMARY, SECONDARY), AND ELASTIC PROPERTIES AS THE TEN RANDOM VARIABLES.



PROB. ANALYSIS - MAT. ORIEN, ELASTIC CONSTANTS AND GEOMETRY AS RANDOM  
 MEAN VALUE - EFFECTIVE STRESS (PSI)



THE CONTOUR PLOTS SHOW THE MEAN VALUE LEVELS OF THE EFFECTIVE STRESS FOR THE  
DETERMINISTIC LOAD CASE.



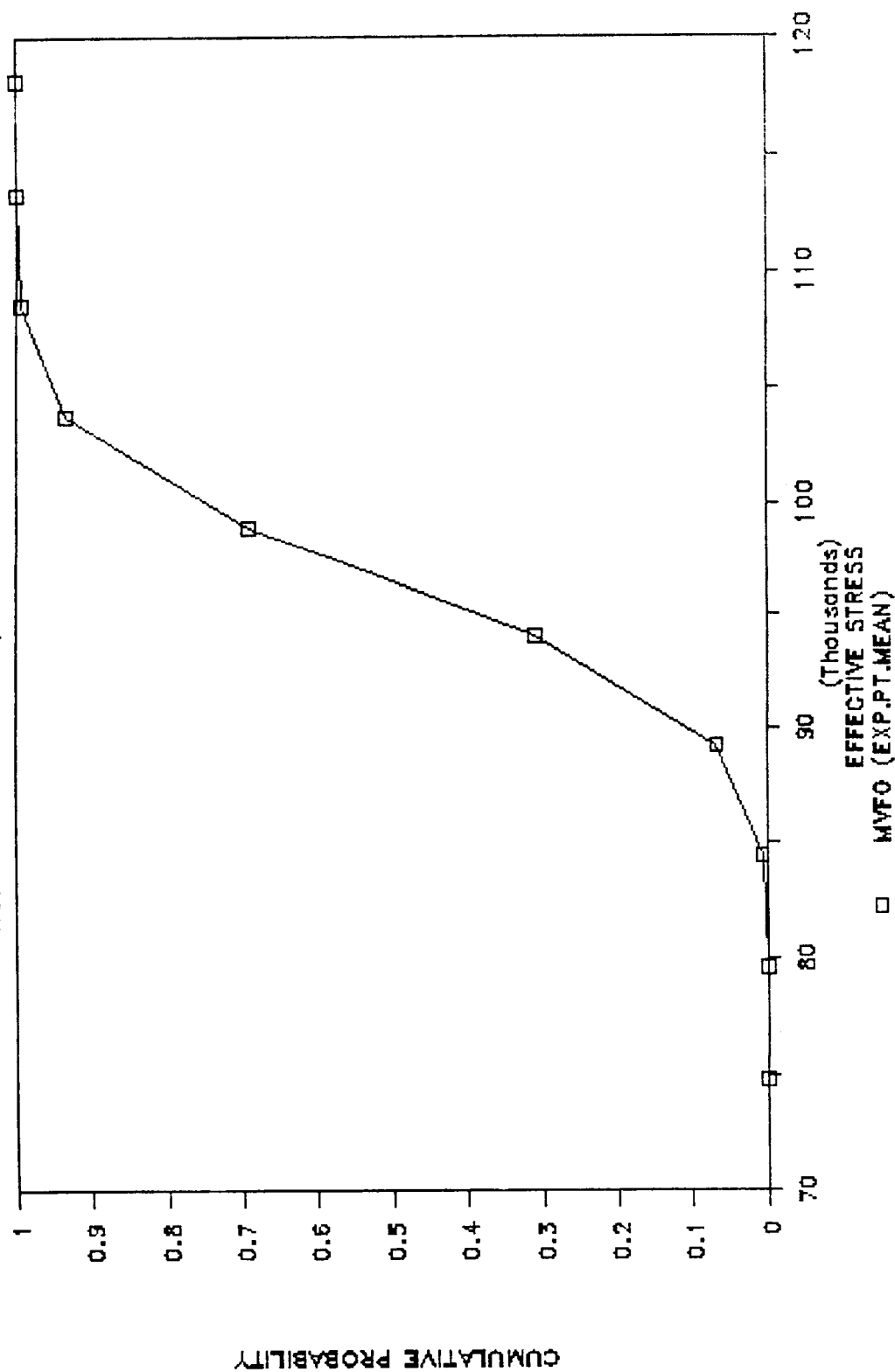
PROB. ANALYSIS - MAT. ORIEN, ELASTIC CONSTANTS AND GEOMETRY AS RANDOM  
 COEFFICIENT OF VARIATION - EFFECTIVE STRESS (PSI)

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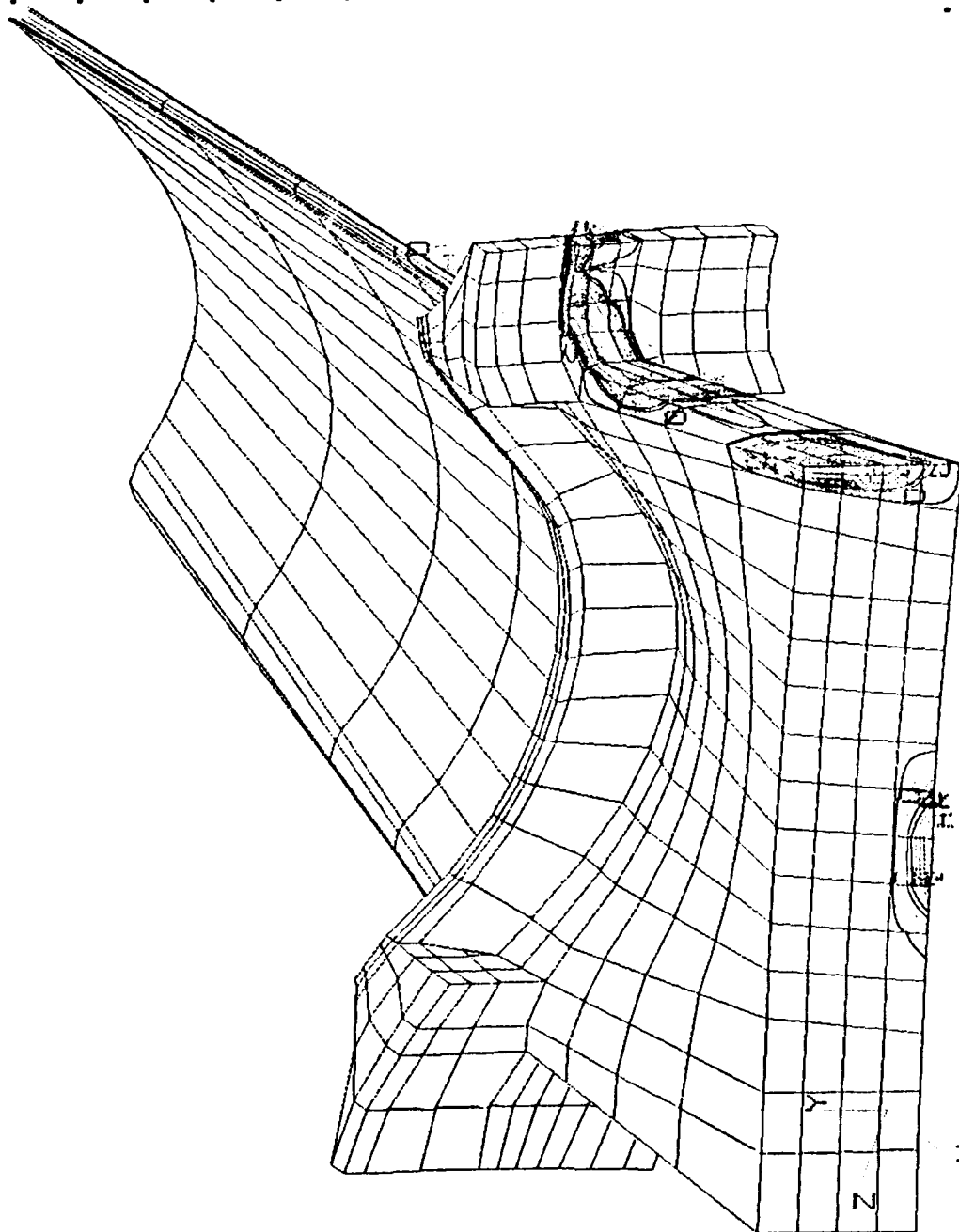
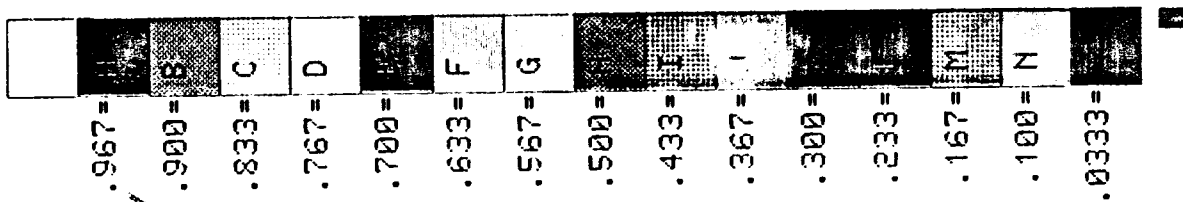
THE COEFFICIENT OF VARIATION CONTOURS ARE PLOTTED FOR THE RANDOM CONDITIONS. THE COEFFICIENT OF VARIATION IS DETERMINED BASED ON THE FIRST-ORDER PERTURBATION ABOUT THE MEAN STATE.

# CUMULATIVE DISTRIBUTION FUNCTIONS

NODE 2518-MAT.PROPS, GEOMETRY RANDOM

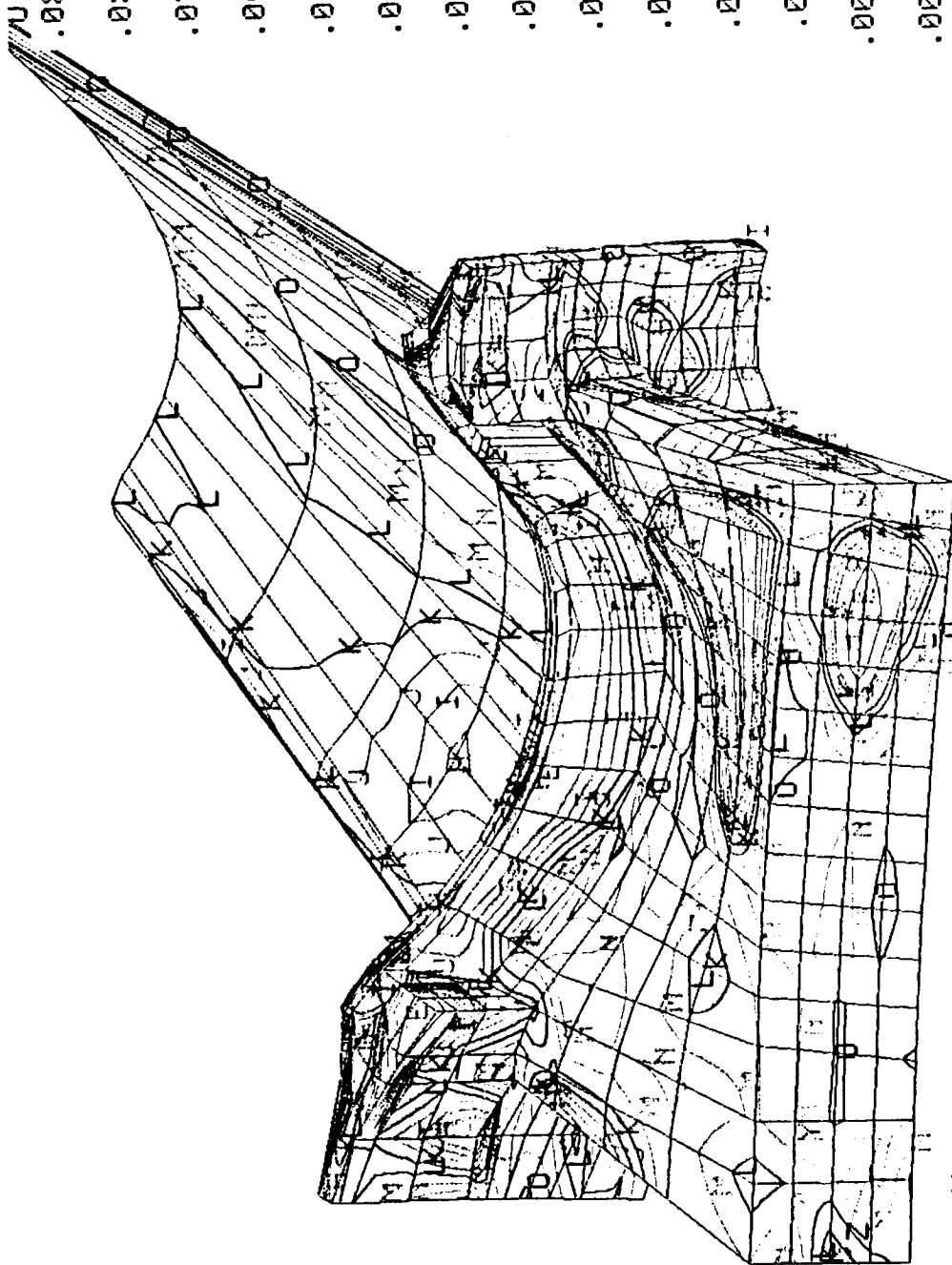
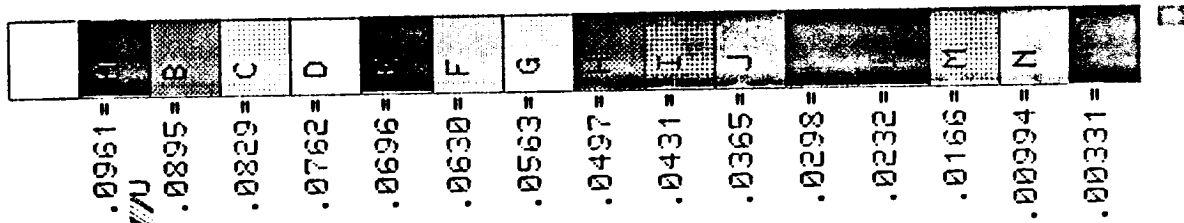


THE CUMULATIVE DISTRIBUTION FUNCTION FOR THE EFFECTIVE STRESS AT NODE 2518 IS  
PLOTTED BASED ON THE FIRST-ORDER PERTURBATION ABOUT THE MEAN STATE.



PROB. ANALYSIS - MAT. ORIENT, ELASTIC CONSTANTS AND GEOMETRY AS RANDOM  
EFFECTIVE STRESS - PROB. OF EXCEEDING 80000 PSI

CUMULATIVE PROBABILITY LEVEL CONTOURS ARE PLOTTED SHOWING THE PROBABILITY OF EXCEEDING 80,000 PSI DUE TO THE DEFINED RANDOM VARIABLES. THREE "HOT-SPOTS" ARE INDICATED.

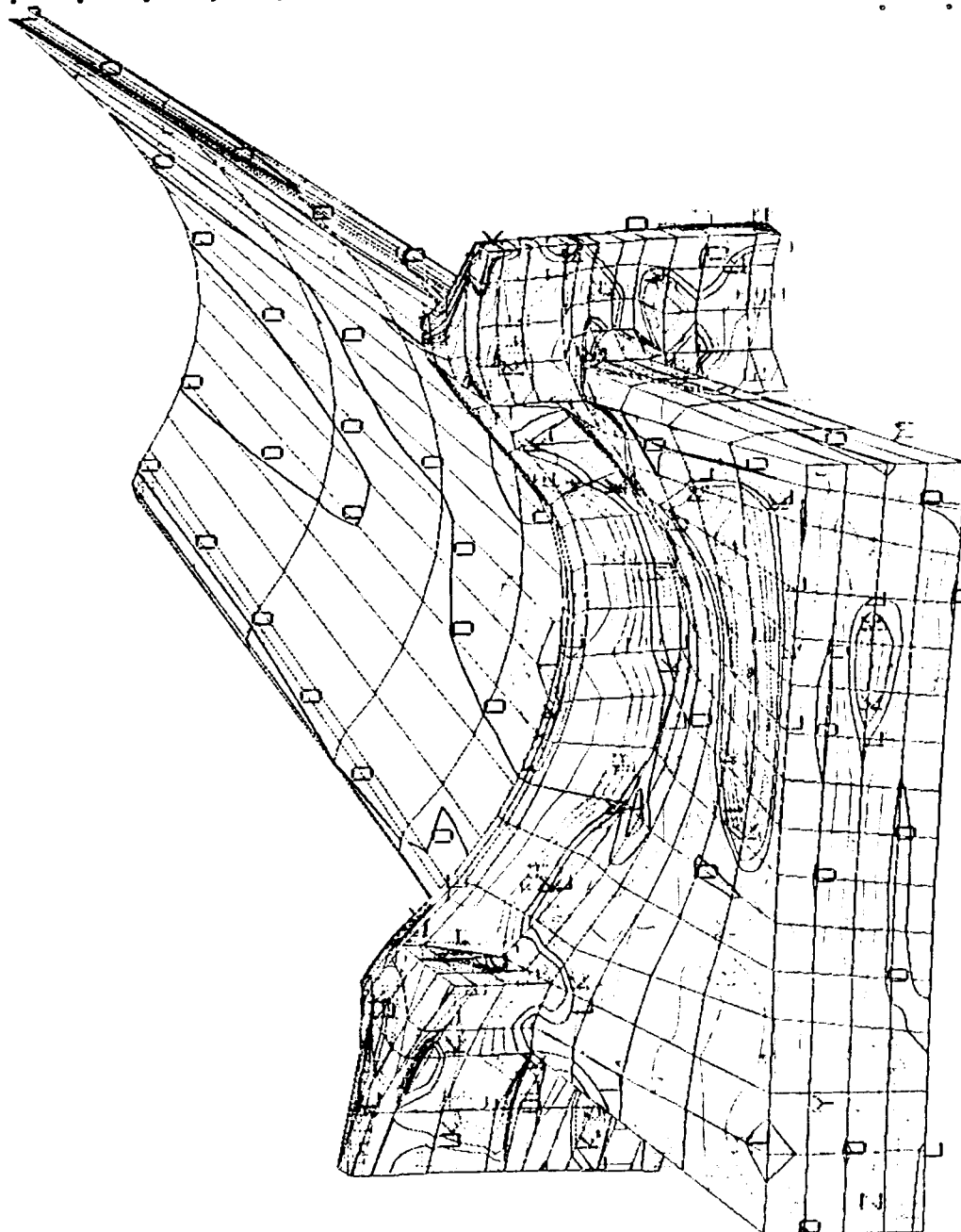
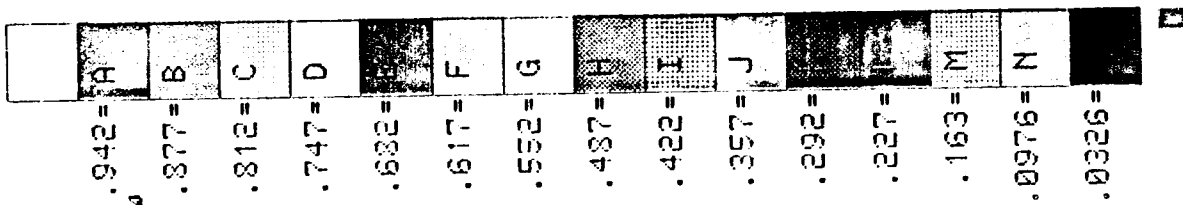


PROB. ANALYSIS - MAT. ORIENT, ELASTIC CONSTANTS AND GEOMETRY AS RANDOM SENSITIVITY FACTOR FOR THE GEOMETRIC TWIST ANGLE

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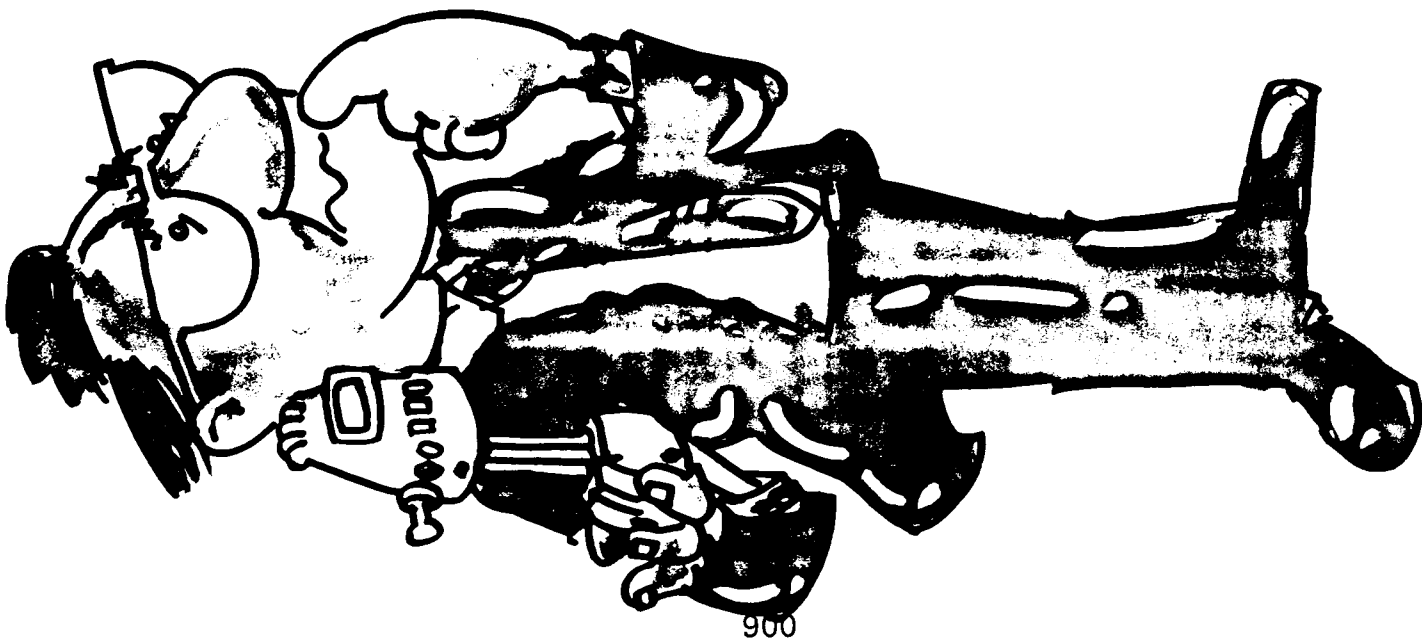


THE CONTOURS INDICATED A RELATIVE MEASURE OF SENSITIVITY OF THE EFFECTIVE STRESS DUE TO THE RANDOM VARIABLE OF GEOMETRIC TWIST OF THE MODEL. THE SENSITIVITY LEVEL MEASURE IS NONDIMENSIONAL AND INCLUDES THE EFFECT OF THE STATISTICAL CHARACTER OF THE VARIABLE AND THE SENSITIVITY OF STRESS TO THE VARIABLE.



PROB. ANALYSIS - MAT. ORIENT, ELASTIC CONSTANTS AND GEOMETRY AS RANDOM  
SENSITIVITY FACTOR - SECON. MAT. ORIENT. (THETAX)

THE CONTOURS OF SENSITIVITY FACTOR FOR EFFECTIVE STRESS DUE TO THE SPECIFIED  
UNCERTAINTY IN THE SECONDARY MATERIAL ORIENTATION ARE SHOWN.



# WHY DO PROBABILISTIC DESIGN?

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- HOW SAFE IS IT?
- HOW WILL IT PERFORM?
- WHAT IS MY CONFIDENCE?
- HOW CAN I MAKE IT MORE RELIABLE?



THE USE OF PROBABILISTIC STRUCTURAL ANALYSIS IS FUNDAMENTAL TO THE ASSESSMENT OF STRUCTURAL PERFORMANCE FOR UNCERTAINTIES IN MATERIAL PROPERTIES, GEOMETRY, BOUNDARY CONDITIONS, AND LOADING. THE NUMERICAL RESULTS CAN BE USED TO DEFINE THE RELATIVE IMPORTANCE OF THE UNCERTAIN VARIABLES, THEREBY PROVIDING DESIGN SUPPORT CAPABILITY FOR MAKING THE STRUCTURE MORE RELIABLE.